KAIST–CHEP–94/04
SNUTP–94–62

K_{Le5} decay as a background in search for $K_L \to \pi^0 \mu^\pm e^\mp$

Pyungwon Ko[a*], Kang Young Lee[b†] and Jae Kwan Kim[b]

[a] *Dept. of Physics, Hong-Ik University, Seoul 121-791, KOREA*

[b] *Dept. of Physics, KAIST, Taejon 305-701, KOREA*

Abstract

We consider a process $K_{Le5}(K_L \to \pi^0\pi^0\pi^\pm e^\mp \nu)$ as a standard model background to the experiment $K_L \to \pi^0\mu^\pm e^\mp$, which seeks for possible violation of lepton family number. Using the lowest order chiral lagrangian, we find that the branching ratio for $K_L \to \pi^0\pi^0\pi^\pm e^\mp \nu$ to be 6.2×10^{-12}. A similar decay $K_L \to \pi^\mp\pi^\mp\pi^\pm e^\pm \nu$ has a branching ratio, 1.7×10^{-11}.

Typeset using REVTEX

*pko@phya.snu.ac.kr

†kylee@chep5.kaist.ac.kr

In the standard model, neutrinos are assumed to be massless so that the individual lepton numbers are exactly conserved separately. Thus, such decays as $K_L \to \mu^\pm e^\mp$ and $K \to \pi \mu^\pm e^\mp$ are not allowed in the standard model (SM). However, this is no longer true in many models beyond SM where neutrinos are endowed with small masses. These small neutrino masses can generate lepton flavor oscillations, lepton family number violations, and may explain the solar neutrino problem and the atmospheric problem simultaneously.

In view of these, it is important to look for lepton family number violating processes such as $K \to \pi \mu^\pm e^\mp$ and $K_L \to \mu^\pm e^\mp$, independently of other neutrino experiments. Up to now, only $K^\pm \to \pi^\pm \mu^\pm e^\mp$ and $K_L \to \mu^\pm e^\mp$ have been searched for :

$$B(K^\pm \to \pi^\pm \mu^\pm e^\mp) < 2.1 \times 10^{-10} \quad (\mathrm{AGS}\ [1]), \tag{1}$$
$$B(K_L \to \mu^\pm e^\mp) < 9.7 \times 10^{-11} \quad (\mathrm{KEK}-137\ [2]), \tag{2}$$
$$3.3 \times 10^{-11} \quad (\mathrm{AGS}-791\ [3]). \tag{3}$$

Another decay $K_L \to \pi^0 \mu^\pm e^\mp$ is being studied at FNAL for the first time. This decay is related with $K^\pm \to \pi^\pm \mu^\pm e^\mp$ via isospin rotation, if the quark family number is unbroken, implying that $B(K_L \to \pi^0 \mu^\pm e^\mp) < 4 \times 10^{-10}$. However, the quark family number may be broken in beyond standard models just as the lepton family number could be broken. Therefore, a search for $K_L \to \pi^0 \mu^\pm e^\mp$ can provide independent informations on the lepton family number violation as a possible signal due to new physics.

Typically, the branching ratio for $K_L \to \mu^\pm e^\mp$ in beyond SM is an order of $\sim O(10^{-18})$ [4]. Thus, the branching ratio for $K_L \to \pi^0 \mu^\pm e^\mp$ should be an order of $\sim O(10^{-19\sim-20})$, because *(i)* the phase space is less and *(ii)* the Nambu–Goldstone nature of a pion produces one more power of the pion momentum in the amplitude compared to $K_L \to \pi^0 \mu^\mp e^\pm$.

The expected branching ratio for $K_L \to \pi^0 \mu^\pm e^\mp$ being so small, one can expect that there would be several sources of backgrounds from SM which can mimic this decay. For example, semileptonic K_{e4} and K_{e5} decays may become backgrounds when a charged pion is misidentified as a muon. However, these possibilities can be easily overcome by imposing a constraint that the invariant mass of the $\pi^\pm e^\mp$ and π^0 (two photons with $m_{\gamma\gamma} = m_{\pi^0}$ in actualty) is around m_K with an experimental uncertainty, typically $m_K \pm 10$ MeV. However, there is an exception for this argument in the case of K_{Le5}. In $K_L \to \pi^0 \pi^0 \pi^\mp e^\pm \nu$, there arise four photons decaying from two π^0's, and two photons (each from different parental π^0) could escape detection and remaining two photons may have the invariant mass around m_{π^0} accidentally. To estimate this kind of background from K_{Le5}, we need to know the amplitude for this process as well as experimental settings. The first decay K_{e4} have been studied in detail both theoretically and experminetally, whereas the K_{Le5} decay has not been discussed in the literature.

In this paper, we study a process K_{e5} as a possible standard model background to an experiment searching for $K_L \to \pi^0 \mu^\pm e^\mp$ using the lowest order chiral perturbation theory. Our amplitude can be used for a background estimate in search for $K_L \to \pi^0 \mu^\mp e^\pm$ as described in the previous paragraph. For completeness, we also include another decay $K_L \to \pi^\mp \pi^\mp \pi^\pm e^\pm \nu$ at the end.

Strong and electromagnetic interactions among pion and kaons and their weak decays are well described in terms of the chiral lagrangian invariant under $SU(3)_L \times SU(3)_R$ transformations as long as their four–momenta are not too large. Since $m_{K_L} = 498$ MeV and $m_\pi = 140$ MeV, each pion momentum is very small in the K_{e5} decay. Hence it is sufficient to

consider the lowest order lagrangian only. At the lowest order $O(p^2)$, the $SU(3)_L \times SU(3)_R$ chiral lagrangian is [5]

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{ Tr } (D_\mu \Sigma D^\mu \Sigma^\dagger) + \frac{f_\pi^2}{2} \text{ Tr } (\mu M \Sigma + \Sigma^\dagger \mu M) \tag{4}$$

where

$$D_\mu \Sigma = \partial_\mu \Sigma - i r_\mu \Sigma + i \Sigma \; l_\mu, \tag{5}$$

$$\Sigma = \exp(\frac{2i}{f_\pi} \phi), \tag{6}$$

$f_\pi = 93$ MeV is the pion decay constant, and ϕ is the pseudoscalar Nambu–Goldstone boson matrix,

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K^0} & -2\eta/\sqrt{6} \end{pmatrix},$$

and $M = \text{diag}(m_u, m_d, m_s)$ is the current quark mass matrix. The matrix field Σ transforms as $\Sigma \to R \Sigma L^\dagger$ under chiral $SU(3)_L \times SU(3)_R$ transformations.

The external gauge fields l_μ, r_μ are appropriate linear combinations of $SU(2)_L \times U(1)_Y$ electroweak gauge fields of the standard model. Since we are not interested in electromagetic interactions in this work, we may set

$$\begin{aligned} l_\mu &= -\frac{g}{\sqrt{2}} W_\mu^- T, \\ r_\mu &= 0, \end{aligned} \tag{7}$$

where

$$T = \begin{pmatrix} 0 \; 0 \; V_{us} \\ 0 \; 0 \; 0 \\ 0 \; 0 \; 0 \end{pmatrix}.$$

The amplitude for $K^0 \to (3\pi)^- e^+ \nu$ can be written as

$$\mathcal{M}(K^0 \to (3\pi)^- l^+ \nu) \equiv \frac{G_F}{\sqrt{2}} \; V_{us} \; \langle (3\pi)^- | \bar{s} \gamma_\mu (1 - \gamma_5) u | K^0 \rangle \; \bar{u}_\nu \gamma^\mu (1 - \gamma_5) v_l. \tag{8}$$

The matrix element of the $(V - A)$ hadronic current

$$H_\mu^{abc}(p_1, p_2, p_3) \equiv \langle \pi^a(p_1) \pi^b(p_2) \pi^c(p_3) | \bar{s} \gamma_\mu (1 - \gamma_5) u | K^0(k) \rangle \tag{9}$$

can be read off from the gauged chiral lagrangian, Eq. (4), as coefficients of $g V_{us} W_\mu^- / 2\sqrt{2}$. In Eq. (9), the superscripts a, b, c denote the electric charge of each pion, and the third pion is the charge odd one as usual. The square of the matrix element summed over the spins of the final leptons is given by (suppressing the superscripts of H_μ for the moment)

$$|\bar{\mathcal{M}}|^2 = \frac{G_F^2}{2}\ |V_{us}|^2\ H_{\rho\sigma}\ L^{\rho\sigma}, \tag{10}$$

where

$$H_{\rho\sigma} = H_\rho H_\sigma^*, \tag{11}$$

$$L_{\rho\sigma} = 8\left[p_{e\rho}p_{\nu\sigma} + p_{\nu\rho}p_{e\sigma} - g_{\rho\sigma}p_e\cdot p_\nu + i\epsilon_{\rho\sigma\alpha\beta}\ p_e^\alpha p_\nu^\beta\right]. \tag{12}$$

Let us first consider $K^0 \to \pi^0\pi^0\pi^- e^+\nu$. It is straightforward to extract from Eq. (4) the following interaction lagrangian relevant to the $K^0 \to \pi^0\pi^0\pi^- e^+\nu$ decay :

$$\begin{aligned}
\mathcal{L}_s = & -\frac{1}{12f_\pi^2}\left(\pi^0\pi^0\partial_\mu K^0\partial^\mu\bar{K}^0 + \bar{K}^0K^0\partial_\mu\pi^0\partial^\mu\pi^0 - \pi^0K^0\partial_\mu\pi^0\partial^\mu\bar{K}^0 - \pi^0\bar{K}^0\partial_\mu\pi^0\partial^\mu K^0\right)\\
& + \frac{\sqrt{2}}{4f_\pi^2}\left(\pi^0K^0\partial_\mu K^-\partial^\mu\pi^+ + \pi^+K^-\partial_\mu\pi^0\partial^\mu K^0 - \pi^0K^-\partial_\mu\pi^+\partial^\mu K^0 - \pi^+K^0\partial_\mu\pi^0\partial^\mu K^-\right)\\
& + \frac{1}{3f_\pi^2}\left(\pi^0\pi^+\partial_\mu\pi^-\partial^\mu\pi^0 + \pi^0\pi^-\partial_\mu\pi^+\partial^\mu\pi^0 - \pi^0\pi^0\partial_\mu\pi^+\partial^\mu\pi^- - \pi^+\pi^-\partial_\mu\pi^0\partial^\mu\pi^0\right)\\
& + \frac{1}{12f_\pi^2}\left(\mu(3m_d+m_s)K^0\bar{K}^0\pi^0\pi^0 + \sqrt{2}\mu(m_u-m_d)K^0K^-\pi^0\pi^+ + 2\mu(m_u+m_d)\pi^0\pi^0\pi^+\pi^-\right)
\end{aligned} \tag{13}$$

$$\begin{aligned}
\mathcal{L}_w = -i\frac{g}{2\sqrt{2}}V_{us}W^{-\mu}&\left[\left(\partial_\mu\pi^+K^0 - \partial_\mu K^0\pi^+\right) + \frac{1}{\sqrt{2}}\left(\partial_\mu\pi^0K^+ - \partial_\mu K^+\pi^0\right)\right.\\
&\left.-\frac{1}{12f_\pi^2}\left(7\partial_\mu\pi^+\pi^0\pi^0K^0 - 6\partial_\mu\pi^0\pi^0\pi^+K^0 - \partial_\mu K^0\pi^0\pi^0\pi^+\right)\right]
\end{aligned} \tag{14}$$

In this work, we ignore isospin symmetry breaking due to $m_u \neq m_d$. Therefore, $\mu(3m_d + m_s) \simeq (m_K^2 + m_\pi^2)$ with $m_K = 498$ MeV, and $m_\pi = 135$ MeV, *etc.*.

Feynman diagrams relevant to the $K^0 \to \pi^0\pi^0\pi^- e^+\nu$ decay derived from the above lagrangians are shown in Fig. 1. The closed circle and the square blob denote strong and weak vertices, respectively. Evaluating the Feynman diagrams shown in Fig. 1, one gets

$$H_\mu^{00-}(p_1,p_2,p_3) = \Sigma_{i=a}^{d}\mathcal{M}_\mu^{(i)}, \tag{15}$$

where

$$\mathcal{M}_\mu^{(a)} = \frac{1}{6f_\pi^2}\ \left[2(p_1+p_2)_\mu - 8p_{3\mu} + L_\mu\right], \tag{16}$$

$$\mathcal{M}_\mu^{(b)} = -\frac{1}{3f_\pi^2}\ \frac{(2q_\mu + L_\mu)}{q^2 - m_\pi^2}\ \left[m_\pi^2 + 2(2p_1\cdot p_2 - p_1\cdot p_3 - p_2\cdot p_3)\right], \tag{17}$$

$$\mathcal{M}_\mu^{(c)} = -\frac{1}{6f_\pi^2}\ \frac{(2p_{3\mu} + L_\mu)}{(k-p_1-p_2)^2 - m_K^2}\ \left[k\cdot(p_1+p_2) + (p_1+p_2)^2\right], \tag{18}$$

$$\mathcal{M}_\mu^{(d)} = -\frac{1}{2f_\pi^2}\ \frac{(2p_{2\mu} + L_\mu)}{(k-p_1-p_3)^2 - m_K^2}\ \left[k\cdot(p_1-p_3)\right]\ + (p_1 \leftrightarrow p_2), \tag{19}$$

with $L = (p_e + p_\nu)$, $q = (p_1 + p_2 + p_3) = (k - L)$. Thus, to the lowest order in chiral expansion, the matrix element of the $(V - A)$ hadronic current in $K^0 \to \pi^0\pi^0\pi^+ e^-\nu$ depend on four independent form factors, F_i $(i = 1, 2, 3, 4)$:

$$H_\mu^{00-} = F_1 p_{1\mu} + F_2 p_{2\mu} + F_3 p_{3\mu} + F_4 L_\mu. \tag{20}$$

In actualty, the last form factor F_4 can be ignored in the following since its contribution is proportional to the lepton mass (m_e in the present case).

In order to get the physical quantities from the amplitude obtained above, we have to perform the 5–body phase space integration. First of all, let us write the differential decay rate for K_{Le5} decay as

$$d\Gamma(K_L \to \pi^0\pi^0\pi^\pm e^\mp \nu) = \frac{1}{2m_K(2\pi)^{11}} \frac{G_F^2}{2} |V_{us}|^2 \ H_{\rho\sigma}^{00\pm} L^{\rho\sigma} d_5(PS), \tag{21}$$

Here, the decay rate includes sum over all possible charge states of the final lepton. In order to take into account the two identical π^0's in the final state, we have to divide the final result obtained from (21) by factor of 2. The five body phase space $d_5(PS)$ is defined as

$$d_5(PS) = \delta^4(k - \Sigma_i p_i) \ \Pi_i \frac{d^3\vec{p}_i}{2E_i}, \tag{22}$$

and can be expressed in terms of products of reduced two body phase space in a standard manner. The five body phase space integration in Eq. (21) was numerically performed, and we get

$$B(K_L \to \pi^0\pi^0\pi^\pm e^\mp \nu) = 6.2 \times 10^{-12}. \tag{23}$$

Being this small, the possibility that K_{Le5} might be a background to $K_L \to \pi^0\mu^\pm e^\mp$ may be negligible at the current stage of experiments. However, one needs more detailed simulations of the actual experiment using our amplitude Eqs. (16)–(19) to be more definite.

For completeness, we consider $K_L \to \pi^+\pi^-\pi^\pm e^\mp \nu$. There are only three Feynman diagrams which contribute to this decay as shown in Figs. 1 (a)–(c) in paretheses. The interaction lagrangian relevant to this decay is

$$\mathcal{L}_w = i\frac{gV_{us}}{12\sqrt{2}f_\pi^2} \ W^{-\mu} \ \left[4\partial_\mu\pi^+ K^0\pi^+\pi^- - 3\partial_\mu\pi^- K^0\pi^+\pi^+ - \partial_\mu K^0\pi^+\pi^+\pi^-\right], \tag{24}$$

as well as Eq. (14), and

$$\begin{aligned}\mathcal{L}_s = \frac{1}{6f_\pi^2} & \left[\pi^-\pi^-\partial_\mu\pi^+\partial^\mu\pi^+ + \pi^+\pi^+\partial_\mu\pi^-\partial^\mu\pi^- - 2\pi^+\pi^-\partial_\mu\pi^+\partial^\mu\pi^- + \mu(m_u+m_d)\pi^+\pi^+\pi^-\pi^-\right] \\ & + \frac{1}{6f_\pi^2} \left[2\left(\pi^+\bar{K}^0\partial_\mu\pi^-\partial^\mu K^0 + \pi^- K^0\partial_\mu\pi^+\partial^\mu\bar{K}^0\right) - \pi^-\bar{K}^0\partial_\mu\pi^+\partial^\mu K^0 - \pi^+ K^0\partial_\mu\pi^-\bar{K}^0\right. \\ & \qquad \left. - \pi^+\pi^-\partial_\mu K^0\partial^\mu\bar{K}^0 - K^0\bar{K}^0\partial_\mu\pi^+\partial^\mu\pi^- + \mu(m_u+2m_d+m_s)K^0\bar{K}^0\pi^+\pi^-\right]\end{aligned} \tag{25}$$

Calculating these diagrams with the interaction lagrangians given in Eqs. (9),(24) and (25), we get

$$H_\mu^{--+}(p_1,p_2,p_3) = \Sigma_{i=a}^c \mathcal{M}_\mu^{(i)}, \tag{26}$$

where

$$\mathcal{M}_\mu^{(a)} = -\frac{1}{3f_\pi^2}\ \left[3(p_1+p_2)_\mu - 2p_{3\mu} + L_\mu\right], \tag{27}$$

$$\mathcal{M}_\mu^{(b)} = -\frac{2}{3f_\pi^2}\ \frac{(2q_\mu + L_\mu)}{q^2 - m_\pi^2}\ \Big[m_\pi^2 - 2p_1\cdot p_2 + p_1\cdot p_2 + p_2\cdot p_3\Big], \tag{28}$$

$$\mathcal{M}_\mu^{(c)} = -\frac{1}{3f_\pi^2}\ \frac{(2p_{1\mu} + L_\mu)}{(p_1+L)^2 - m_K^2}\ \Big[m_\pi^2 + p_1\cdot p_3 + k\cdot(2p_3 - p_1)\Big] + (p_1 \leftrightarrow p_2). \tag{29}$$

Summing over the all possible charges of the final lepton and performing the phase space integrations numerically, we get

$$B(K_L \to \pi^\pm\pi^\pm\pi^\mp e^\mp\nu) = 1.7\times 10^{-11}. \tag{30}$$

In conclusion, the amplitudes for K_{Le5} decays are derived using the lowest order chiral perturbation theory, which can be used by experimentalists in order to study the background to $K_L \to \pi^0\mu^\pm e^\mp$ coming from the decay, $K_L \to \pi^0\pi^0\pi^\pm e^\mp\nu$. The resulting branching ratio for a decay $K_L \to \pi^0\pi^0\pi^\pm e^\mp\nu$ is about 6.2×10^{-12}. The branching ratio for a similar decay $K_L \to \pi^\mp\pi^\mp\pi^\pm e^\pm\nu$ is twice larger, 1.7×10^{-11}. Thus, these decays are unlikely to be observed in the current and near-future experiments.

ACKNOWLEDGMENTS

P.K. thanks Prof. J.L. Rosner for suggesting this problem and useful correspondence. We are also grateful to Prof. S.K. Kim for discussions about the experimental situations. P.K. was supported in part by KOSEF through CTP at Seoul National University, and by the Basic Science Research Institute Program, Ministry of Education, 1994, Project No. BSRI–94–2425. K.Y.Lee and J.K. Kim were supported by KOSEF.

REFERENCES

[1] A.M. Lee *et al.*, Phys. Rev. Lett. **64**, 165 (1990).
[2] T. Agaki *et al.*, Phys. Rev. Lett. **67**, 2614 (1991).
[3] K. Arisaka *et al.*, Phys. Rev. Lett. **70**, 1049 (1993).
[4] W. Marciano and A. Sanda, Phys. Rev. Lett. **38**, 1512 (1977) ; W. Marciano, Phys. Rev. **D 45**, R721 (1992). Phys. Rev. **D 38**, 2841 (1988).
[5] S. Weinberg, Physica **96A**, 327 (1979) ; J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984) ; J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).

FIGURES

FIG. 1. Feynman diagrams relevant to $K^0 \to \pi^0\pi^0\pi^- e^+\nu$ in chiral perturbation theory to $O(p^2)$. The square blob and the closed circle represent strong and weak interaction vertices, respectively. (The diagrams for $K^0 \to \pi^-\pi^-\pi^+ e^+\nu$ in he parentheses in (a), (b) and (c). There is no diagram anologous to (d) contributing to $K^0 \to \pi^-\pi^-\pi^+ e^+\nu$.)